MATH 1650 PROPERTIES OF LOGARITHMS

EXAMPLE: Use the properties of logarithms to write the following as a single logarithm.

1. $\log_3(3^x) = x$
2. $\ln\left(e^{0.01t}\right) = \log_e\left(e^{0.01t}\right) = 0.01t$
3. $10^{\log(-0.2)} = 10^{\log_{10}(-0.2)} = -0.2$
4. $e^{x\ln(2)} = e^{\ln(2^x)} = e^{\log_e(2^x)} = 2^x$

EXAMPLE: 'Expand' the following using the properties of logarithms and simplify. That is, rewrite products as sums, quotients as differences, and powers as factors.

Assume when necessary that all quantities represent positive real numbers.

1.
$$\begin{aligned}
\log_2\left(\frac{8}{x}\right) &= \log_2(8) - \log_2(x) && \text{Quotient Rule} \\
&= 3 - \log_2(x) && \text{Since } 2^3 = 8 \\
&= -\log_2(x) + 3
\end{aligned}$$

2.
$$\begin{aligned}
\log_{0.1}\left(10x^2\right) &= \log_{0.1}(10) + \log_{0.1}\left(x^2\right) && \text{Product Rule} \\
&= \log_{0.1}(10) + 2\log_{0.1}(x) && \text{Power Rule} \\
&= -1 + 2\log_{0.1}(x) && \text{Since } (0.1)^{-1} = 10 \\
&= 2\log_{0.1}(x) - 1
\end{aligned}$$

3.
$$\begin{aligned}
\ln\left(\frac{3}{et}\right)^2 &= 2\ln\left(\frac{3}{et}\right) && \text{Power Rule} \\
&= 2\left[\ln(3) - \ln(et)\right] && \text{Quotient Rule} \\
&= 2\ln(3) - 2\ln(et) \\
&= 2\ln(3) - 2\left[\ln(e) + \ln(t)\right] && \text{Product Rule} \\
&= 2\ln(3) - 2\ln(e) - 2\ln(t) \\
&= 2\ln(3) - 2 - 2\ln(t) && \text{Since } e^1 = e \\
&= -2\ln(t) + 2\ln(3) - 2
\end{aligned}$$

4.

$$\begin{aligned}
\log \sqrt[3]{\frac{100x^2}{yz^5}} &= \log\left(\frac{100x^2}{yz^5}\right)^{1/3} \\
&= \frac{1}{3}\log\left(\frac{100x^2}{yz^5}\right) && \text{Power Rule} \\
&= \frac{1}{3}\left[\log\left(100x^2\right) - \log\left(yz^5\right)\right] && \text{Quotient Rule} \\
&= \frac{1}{3}\log\left(100x^2\right) - \frac{1}{3}\log\left(yz^5\right) \\
&= \frac{1}{3}\left[\log(100) + \log\left(x^2\right)\right] - \frac{1}{3}\left[\log(y) + \log\left(z^5\right)\right] && \text{Product Rule} \\
&= \frac{1}{3}\log(100) + \frac{1}{3}\log\left(x^2\right) - \frac{1}{3}\log(y) - \frac{1}{3}\log\left(z^5\right) \\
&= \frac{1}{3}\log(100) + \frac{2}{3}\log(x) - \frac{1}{3}\log(y) - \frac{5}{3}\log(z) && \text{Power Rule} \\
&= \frac{2}{3} + \frac{2}{3}\log(x) - \frac{1}{3}\log(y) - \frac{5}{3}\log(z) && \text{Since } 10^2 = 100 \\
&= \frac{2}{3}\log(x) - \frac{1}{3}\log(y) - \frac{5}{3}\log(z) + \frac{2}{3}
\end{aligned}$$

5.

$$\begin{aligned}
\log_{117}\left(x^2 - 4\right) &= \log_{117}\left[(x+2)(x-2)\right] && \text{Factor} \\
&= \log_{117}(x+2) + \log_{117}(x-2) && \text{Product Rule}
\end{aligned}$$

EXAMPLE: Use the properties of logarithms to write the following as a single logarithm.

1.

$$\log_3(x-1) - \log_3(x+1) = \log_3\left(\frac{x-1}{x+1}\right)$$

2.

$$\begin{aligned}
\log(x) + 2\log(y) - \log(z) &= \log(x) + \log\left(y^2\right) - \log(z) && \text{Power Rule} \\
&= \log\left(xy^2\right) - \log(z) && \text{Product Rule} \\
&= \log\left(\frac{xy^2}{z}\right) && \text{Quotient Rule}
\end{aligned}$$

3.

$$\begin{aligned}
4\log_2(x) + 3 &= \log_2\left(x^4\right) + 3 && \text{Power Rule} \\
&= \log_2\left(x^4\right) + \log_2\left(2^3\right) && \text{Since } 3 = \log_2\left(2^3\right) \\
&= \log_2\left(x^4\right) + \log_2(8) \\
&= \log_2\left(8x^4\right) && \text{Product Rule}
\end{aligned}$$

4.

$$\begin{aligned}
-\ln(t) - \tfrac{1}{2} &= (-1)\ln(t) - \tfrac{1}{2} \\
&= \ln\left(t^{-1}\right) - \tfrac{1}{2} && \text{Power Rule} \\
&= \ln\left(t^{-1}\right) - \ln\left(e^{1/2}\right) && \text{Since } \tfrac{1}{2} = \ln\left(e^{1/2}\right) \\
&= \ln\left(t^{-1}\right) - \ln\left(\sqrt{e}\right) \\
&= \ln\left(\frac{t^{-1}}{\sqrt{e}}\right) && \text{Quotient Rule} \\
&= \ln\left(\frac{1}{t\sqrt{e}}\right)
\end{aligned}$$

EXAMPLE: Use an appropriate change of base formula to convert the following expressions to ones with the indicated base. Verify your answers using a graphing utility, as appropriate.

1. $3^2 = 10^{2\log(3)}$

2. $10^x = e^{x\ln(10)}$

3. $\log_4(5) = \dfrac{\ln(5)}{\ln(4)}$

4. $\ln(x) = \log_e(x) = \dfrac{\log(x)}{\log(e)}$